Game Theory: Final Exam Review

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What's the Goal of Game Theory?

- Predict the outcomes of **games**.
- **Game:** strategic situation where the results you care about depend on your actions and the actions of others.
- Defined by **payoffs** and **strategies**.
- Example:

	L	C	R
T	$-1, -1$	$7, -1$	$-1, 7$
M	$1, 0$	$2, 5$	$4, -1$
B	$0, 1$	$1, 6$	$3, 2$

First Idea: Delete Dominated Strategies

- If my payoff from playing Strategy A is larger than from playing Strategy B, *regardless of what others do*, then A **dominates** B.
- Can be **strict** or **weak**.
- Examples from previous game: M strictly dominates B. C weakly dominates L.
- Idea: It doesn't make sense to play a dominated strategy. Reduce the game by deleting dominated strategies. Do this iteratively until we can't anymore.
- Problem 1: Not always possible to reduce the game much.
- Problem 2: Iterative deletion of *weakly* dominated strategies can give different answers depending on order of deletion!

Better Idea: Focus on Best Responses

- I have some belief about others' strategies. A **best response** is a strategy that maximizes my expected payoff given that belief.
- Idea: It makes sense to assume people play best responses. Delete strategies that are never best responses to any belief.
- We can go even further: It makes sense to assume people play *correct* best responses.
- **Nash Equilibrium (NE)**: every player plays a best response to the other players' strategies.
- In other words, no player can do strictly better by a unilateral deviation.
- Tip: Iterative deletion of strictly dominated and never-best-response strategies keeps NE the same! Try to reduce games before looking for NE.

Mixed Strategies

- I can play a **mixed strategy** by randomizing over pure strategies.
- If a mixed strategy is a best response, then all pure strategies in the mix must also be best responses (so I'm indifferent across them).
- **Trick to find mixed NE**: First, make all players indifferent across the pure strategies included in their mixes. Second, check that no players can do strictly better by deviating to pure strategies outside of their mixes.
- Example:

		Bob: l	Bob: r
Alice	L	30, 70	80, 20
Alice	R	90, 10	20, 80

Prediction (mixed NE): Alice plays L with probability $\frac{7}{12}$ and Bob plays l with probability $\frac{1}{2}$.

Application: Evolutionary Games

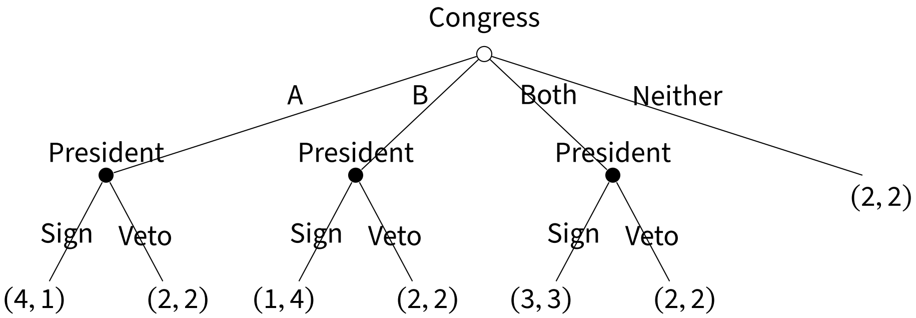
- Two-player symmetric games.
- Find NE where both players play the same strategy. Call that the **incumbent** strategy.
- We can look for **strict NE**: any mutant does *strictly* worse against incumbents than an incumbent would. Interpretation: any mutants die out.
- Or we can look for a slightly weaker condition, **evolutionary stability**: it's possible some Mutant X does *equally* as well against incumbents as an incumbent would. But it does strictly worse against other Mutant X's than an incumbent would.
- Interpretation: any mutants either die out or at least can't get too big.

Sequential Games with Perfect Information

- **Perfect information**: at every **decision node**, the player whose turn it is knows exactly which node they're at.
- A **strategy** is a complete plan of action: what action I take at every node (including nodes that may never be reached!).
- **Subgame:** starting from a singleton node, includes all successor nodes.
- Problem: NE of the whole game may involve players having unrealistic, non-NE beliefs about subgames that are never reached.
- Stronger condition than NE, **subgame perfect equilibrium (SPE):** every subgame is a NE.
- Trick to find SPE: backward induction.

Example 1: Veto Points

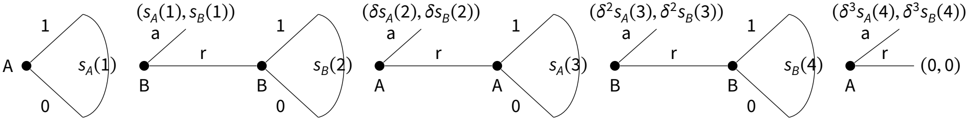
- Congress can pass a bill that includes Proposal A, Proposal B, both, or neither. If they propose anything, the president can then sign or veto it. Their payoffs are below.



- Prediction (SPE): Congress plays Both, president plays (Veto, Sign, Sign).
- Example of NE that is not SPE: Congress plays Neither, president plays (Veto, Veto, Veto).

Example 2: Bargaining

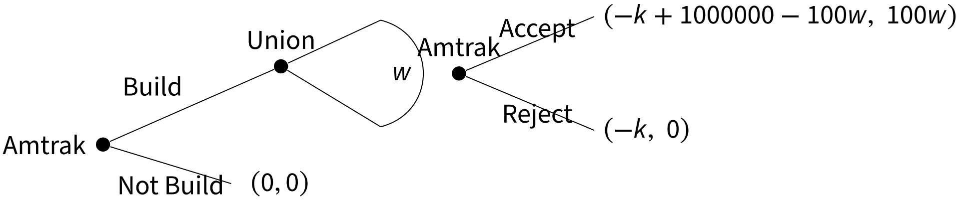
- On Day 1, Alice makes Bob an offer (s_A, s_B) where $s_A + s_B = 1$. If Bob accepts, Alice gets s_A and Bob gets s_B. If Bob refuses, they move on. On Day 2, Bob makes Alice an offer. This repeats for four days. On Day 4, if Alice refuses, no one gets anything. For both, 1 unit tomorrow is worth $\delta < 1$ today.



- Prediction: Day 1 offer is accepted, and allocation is $\left(1 - \delta + \delta^2 - \delta^3, \delta - \delta^2 + \delta^3\right)$.

Example 3: Hold-up Problem

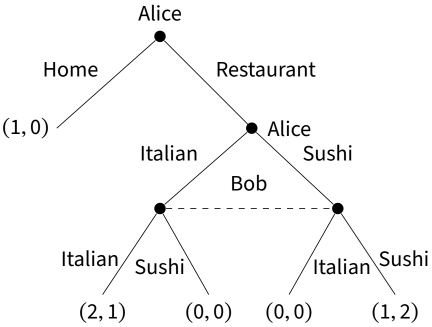
- Amtrak is choosing whether or not to build a railroad. Building the railroad involves an up-front sunk cost k. The railroad will generate $1,000,000 in revenue and employ 100 unionized workers. If the railroad is built, the union can then make a "take it or leave it" wage demand w. Amtrak must accept the wage demand or close the railroad down.



- Prediction: Amtrak will not build.

Sequential Games with Imperfect Information

- **Imperfect information:** the player whose turn it is may not know exactly which node they're at.
- Replace nodes with **information sets** (which group together indistinguishable nodes).
- Example:



		Bob: Italian	Bob: Sushi
Alice	Italian	2, 1	0, 0
	Sushi	0, 0	1, 2

The subgame after Alice chooses Restaurant is equivalent to the matrix on the right.

Incomplete Information

- Even less information than before, **incomplete information:** players may not the rules of the game, such as each others' payoffs or even their own payoffs.
- Example 1: costless signaling of verifiable information. Restaurant safety grades are unknown, but they can choose to show their health certificates.
- **Informational unraveling:** suppose C is average. B's reveal to not be thought of as C's. A's reveal to not be thought of as B's. In the end, all information is revealed.
- Example 2: costly signaling. Workers can't reveal their quality to employers, but they can choose to get a degree that's easier to earn if you're high-quality.
- **Separating equilibrium:** if the degree is costly and rewarding enough, high-quality workers get it and low-quality workers don't.